Random Turn-On Jitter of a Single-Mode Laser – Modeling and Measurements

Przemysław Krehlik*

Institute of Electronics, AGH University of Science and Technology, Mickiewicza 30, 30-059 Kraków, Poland

Received January 5, 2009; accepted January 22, 2009; published March 31, 2009

Abstract— The paper presents experimental verification of Oberman's formula describing a random jitter occurring when a subthreshold biased laser is turned on. The measurements were focused on telecommunication-grade MQW DFB (Multi-Quantum-Well, Distributed Feedback) lasers. For precise characterization of the jitter statistics, 48 hour measurements collecting 200•millions events were performed. It is found that Oberman's formula, even though derived with some questionable simplifications, agrees very well with the experimental data. Particularly, a very good description of probability distribution "tails" makes the formula useful for bit-error-rate calculations.

In standard IM-DD (intensity modulation, direct detection) fiber optic data transmission systems, directly modulated lasers (DMLs) are often used as optical signal sources. The recent advance in high-speed DMLs fabrication allows using them in systems operating at very high data rates up to 10 Gb/s. A laser driver is usually arranged in such a way as to ensure above the threshold laser operation, even in the low (logic "zero") state. In some cases, however, a subthreshold bias (undesired or desired) may occur. Unexpected subthreshold biasing may result from laser aging or temperature increase which pushes a laser threshold current beyond the range accommodated by the driver. Intentionally, subthreshold biasing may be considered for transmitter power consumption saving and/or for enhancing the extinction ratio (ER), which is desired in multipoint-to-point transmission schemes, such as Passive Optical Networks (PONs) [1, 2].

When a current pulse (representing the logic symbol "one") is applied to a laser biased below the threshold, the resulting optical pulse is significantly delayed, which is usually referred to as the turn-on delay. The delay may be divided into two parts: the first (usually dominant) one connected with a relatively slow process of building the carrier density in an active region up to the threshold value, at witch lasing begins. The second one is the time period in which the photon population rapidly builds up.

The first part of the delay depends not only on laser parameters and a driving current at both low and high states but also on the gap between consecutive current pulses. Thus, in data transmission the turn-on delay depends on the number of "zero" symbols preceding the actual current pulse (i.e. the symbol "one") [3]. These turn-on variations are referred to as a deterministic (or pattern dependent) turn-on jitter. This part of the turn-on delay, and even the above mentioned deterministic jitter may be successfully reduced by proper advancing of current pulses [1, 2, 4].

The second part of the turn-on delay, concerned with the photon population growth is strongly affected by the random nature of spontaneous emission, constituting the initial photon population at the beginning of the lasing. In consequence, this delay is also random, thus a so-called random turn-on jitter arises. This jitter may affect the transmission system bit-error-rate (BER), so knowing an adequate description of its probability density function (pdf) is crucial for system modeling.

In the present paper we first present the formula for a random turn-on jitter, very close to the one proposed by Obermann *et al.* in [5]. However, the original paper by Obermann (and some related work such as [6]) gives only coarse experimental verification of the formula. Therefore the main aim of this paper is to present more precise verification, especially in the region of probability distribution "tails", important in BER analysis.

The jitter pdf formula presented here is based on the assumption proposed in [6], that the evolution of photon density may be (with some simplification) divided into two qualitatively different phases. In the first one, when the carrier density is still below the threshold value, the photon density is small and fluctuates randomly due to spontaneous emission. In the second, started when the carrier density reaches the threshold level, the photon population grows rapidly in the deterministic manner. The random photon density at the end of the first phase constitutes, in this treatment, an initial condition for later deterministic growth – see Fig. 1.

The random photon density in the first phase may be described by the exponential pdf as [7]:

* E-mail: krehlik@agh.edu.pl

$$p_S(S) = \frac{1}{\langle S \rangle} \exp\left(-\frac{S}{\langle S \rangle} \right), \quad (1)$$

where S is photon density and $\langle S \rangle$ is its expected value.

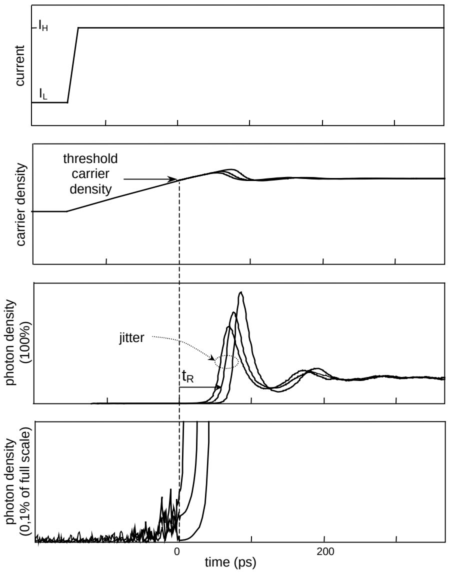

Fig. 1. Illustration of the laser turn-on. In the last plot, an assumed demarcation is shown between random and deterministic photon density evolution.

The growth of photon density in the second phase may be, after some simplification, derived from standard rate equations as:

$$S = S_0 \exp\left(\frac{\Gamma g_0}{2} \frac{I_H - I_{TH}}{eV_a} t^2 \right), \quad (2)$$

where S_0 is an initial photon density at the moment when carrier density reaches the threshold level, Γ is an optical confinement factor, g_0 is a differential gain, e is an electron charge, V_a is an active region volume, I_H is a current pulse amplitude (see Fig. 1), and I_{TH} is a threshold current. Using: $\frac{I - I_{TH}}{eV_a} = \frac{S_H}{\Gamma \tau_P}$ and $\omega_r^2 = \frac{g_0 S_H}{\tau_P}$ (where S_H is a steady-stay photon density corresponding to I_H, ω_r is an angular frequency of relaxation oscillations at I_H, and τ_P is the photon lifetime [7]), (2) may be rewritten as:

$$S = S_0 \exp\left(\frac{\omega_r^2}{2} t^2 \right). \quad (3)$$

Thus, the time needed for a photon density (and also the optical output power) to obtain a half of its steady-stay value is:

$$t_R = \frac{1}{\omega_r} \sqrt{2 \ln\left(\frac{S_H}{2S_0} \right)}. \quad (4)$$

Taking (1) as the pdf of the initial photon density S_0, the pdf of t_R may be calculated as:

$$p_{t_R}(t) = \omega_r^2 t \frac{S_H}{2\langle S_0 \rangle} \exp\left(-\frac{(\omega_r t)^2}{2} \right) \cdot \exp\left(-\frac{S_H}{2\langle S_0 \rangle} \exp\left(-\frac{(\omega_r t)^2}{2} \right) \right) \quad (5)$$

for $t \geq 0$, and

$p_{t_R}(t) = 0$ for $t < 0$.

The main advantage of the above formula is that it describes the pdf of an optical signal increase by using only two parameters: an easy-to- measure frequency of relaxation oscillations ω_r and the ratio: $S_H / \langle S_0 \rangle$, which may be determined, for instance, by matching the measured standard deviation of a turn-on random jitter with that calculated from (5). Additionally, it may be noticed that the ratio $S_H / \langle S_0 \rangle$ has a minor influence on the probability distribution described by (5); changing $S_H / \langle S_0 \rangle$ in the range of 100 to 1000 gives only a 17% difference in the pdf standard deviation.

The experimental setup built to test the usefulness of (5) for turn-on jitter modeling is shown in Fig. 2.

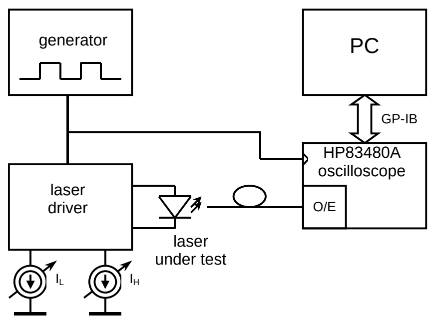

Fig. 2. Measurement setup.

The laser under test was connected to an ultra-fast driver based on the MAX3941 integrated modulator characterized by 23 ps rise/fall time and 0.3 ps rms jitter. The laser optical output was observed with a sampling oscilloscope of 30 GHz bandwidth optical input. The measurement data were collected by a PC computer connected to the oscilloscope. Fig. 3 shows a random turn-on jitter observed on the oscilloscope.

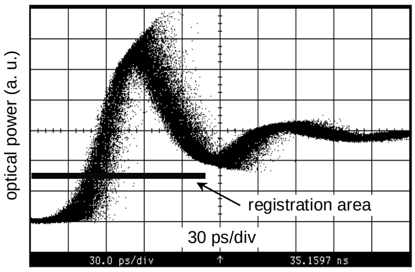

Fig. 3. Oscillogram of the laser turn-on.

Interesting jitter statistics was obtained by gathering the histograms of time positions where the signal reaches the value equal to the half of the steady-state optical power - the corresponding registration area is marked in Fig. 3. The obtained results are exemplified in Fig. 4. The interesting jitter is understood as the difference: $t_R - \langle t_R \rangle$. The laser under study was a MQW DFB 1.55 μm one (type PT3563 by Photon). The f_r = 9.8 GHz taken from the measurement was put into (5), and $S_H / \langle S_0 \rangle = 400$ was matched to achieve the best accuracy.

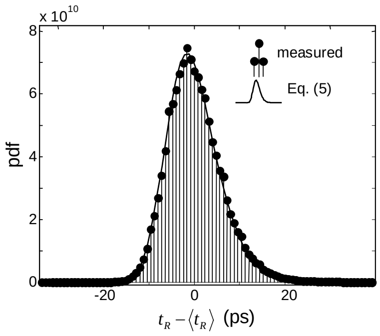

Fig. 4. Comparison of measured jitter histogram and the pdf calculated from (5). The measured histogram contains 10^6 events.

One may observe a good general agreement of measurement with the pdf described by (5).

However, for adequate calculating of the BER resulting from the jitter, it is crucial to verify the accuracy of probability distribution "tails", especially those describing the probability of higher values of the turn-on delay. From this viewpoint it is better to examine not just the pdf, but rather the cumulative distribution function (cdf) and the complementary cumulative distribution function (ccdf). Additionally, a very large number of events should be collected for accurate estimation of the probability of rare events (i.e. the smallest and the highest values of the delay). In Fig. 5 the jitter cdf and ccdf obtained from $200 \cdot 10^6$ registered events are compared to distributions calculated by integrating (5). It may be concluded that the proposed formula is valid for very rare events as well, and so it may be used for BER calculations. For the comparison the cdf and ccdf calculated from the Gaussian pdf having the same standard deviation is also shown in the same figure. As may be noticed, this standard distribution, usually taken as a first approximation of any bell-like pdf, is completely inadequate in our case.

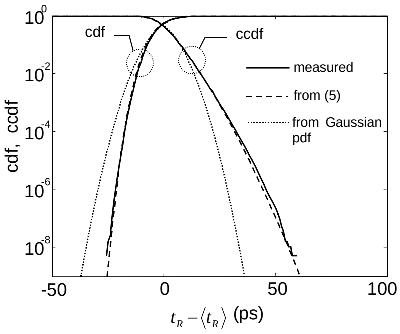

Fig. 5. Jitter cdf and ccdf – obtained from measurement, from (5), and from Gaussian pdf.

Some practical problem in the undertaken measurements was the long-term wander of signal propagation delays affecting the horizontal position of an optical pulse visible on the oscilloscope screen. (Probably the wander was caused by the ambient temperature fluctuations influencing the measurement setup.) Because the measurements collecting $200 \cdot 10^6$ events take in our setup 48 hours, the wander results in noticeable spreading of the measured pdf. To overcome this problem partial histograms were collected four times per hour, then all of them were time domain shifted to have a zero mean value and next they were summed to build up the final histogram.

The same measurements were performed for some other MQW DFB lasers operating in 1.3 μm and 1.55 μm windows. Putting into (5) the measured value of ω_r and $S_H / \langle S_0 \rangle$ in the range of 300 to 600 a similar agreement with a measured jitter was obtained.

References

[1] E. Sackinger, Y. Ota, T. J. Gabara, and W. C. Fischer, IEEE Journal of Solid-State Circuits **35**, 269 (2000).

[2] M. Yano, K. Yamaguchi, and H. Yamashita, FUJITSU Scientific and Technical Journal **35**, 56 (1999).

[3] P. Krehlik and Ł. Śliwczyński, Opto-Electronics Review **12**, 187 (2004).

[4] P. Krehlik and Ł. Śliwczyński, AEÜ International Journal of Electronics and Communications **60**, 331(2006).

[5] K. Obermann, S. Kindt, and K. Petermann, IEEE Photonics Technology Letters **8**, 746 (1996).

[6] L. Zei, K. Obermann, T. Czogalla, and K. Petermann, IEEE Photonics Technology Letters **11**, 6 (1999).

[7] K. Petermann, *Laser diode modulation and noise*, Klüwer, Dordrecht, 1991.

