Giant cross-Kerr nonlinearity in a four-level Y-type atomic system

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Abstract—We found the analytical expression for the cross-Kerr nonlinear coefficient in a four-level Y-type atomic system. The analytical model is applied to ^{85}Rb atoms and shows that the cross-Kerr nonlinear coefficient is enhanced by several orders of magnitude under electromagnetically induced transparency. At the same time, the amplitude and the sign of the cross-Kerr nonlinear coefficient are controlled with respect to the intensity and frequency of the coupling laser field. The analytical model can be helpful to explain the experimental results and to study effects in nonlinear optics.

The cross-Kerr nonlinear effect, i.e., cross-phase modulation (XPM) is the phenomenon in which the phase of an optical field is modulated by another field. It has beneficial applications in quantum phase gate [1], quantum entanglement state [2], all-optical switching [3], and so on. In such applications, significant nonlinearity is required to achieve nonlinear optical processes with low light intensity.

Today, using electromagnetically induced transparency (EIT) [4], one can get large Kerr nonlinearity in the vicinity of atomic resonance with minimal absorption. Indeed, Schmidt *et al.* [5] showed giant cross-Kerr nonlinearity in a four-level N-type atomic system under the EIT condition. And then, Kang *et al.* [6] experimentally verified this model for the Rb atomic medium. Since then, several schemes have been extensively studied theoretically and experimentally to obtain giant nonlinearity in the four-level atomic systems consisting of inverted Y-type [7], tripod-type [8], M-type [9], and double lambda-type [10] configurations.

Recently, we have developed an analytical model for multi-window EIT in a five-level cascade system [11-13] to study the enhancement and control of Kerr nonlinearity [14, 15]. The model has also been used to study applications such as optical bistability [16], controlling group velocity [17, 18].

In this paper, we propose the analytical model of the four-level Y-type atom to study the enhancement and control of the cross-Kerr nonlinear coefficient. The influence of intensity and frequency of coupling field on the cross-Kerr nonlinear coefficient is investigated.

Figure 1 shows the energy-level diagram for the four-level Y-type atomic system. The $|1\rangle\leftrightarrow|2\rangle$ transition is applied by the probe laser beam with frequency ω_p, the

$|2\rangle\leftrightarrow|3\rangle$ transition is coupled by the coupling laser beam with frequency ω_c and the $|2\rangle\leftrightarrow|4\rangle$ transition is driven by the signal laser beam with frequency ω_s. In this configuration, the coupling beam can generate the EIT effect for the probe beam, while the signal beam can induce the cross-Kerr effect for the probe beam.
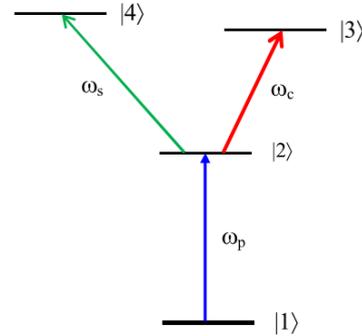
Fig. 1. Energy-level diagram for the four-level Y-type atomic system.

The density matrix equations of motion for the four-level Y-type atoms in the laser beams are given by:

$$\dot{\rho}_{44} = -\left(\Gamma_{43} + \Gamma_{42}\right)\rho_{44} + \frac{i}{2}\Omega_s(\rho_{42} - \rho_{24}), \tag{1}$$

$$\dot{\rho}_{33} = \Gamma_{43}\rho_{44} - \Gamma_{32}\rho_{33} + \frac{i}{2}\Omega_c(\rho_{32} - \rho_{23}), \tag{2}$$

$$\dot{\rho}_{22} = \Gamma_{42}\rho_{44} + \Gamma_{32}\rho_{33} - \Gamma_{21}\rho_{22} + \frac{i}{2}\Omega_p(\rho_{21} - \rho_{12})$$
$$+ \frac{i}{2}\Omega_c(\rho_{23} - \rho_{32}) + \frac{i}{2}\Omega_s(\rho_{24} - \rho_{42}) \tag{3}$$

$$\dot{\rho}_{11} = \Gamma_{21}\rho_{22} + \frac{i}{2}\Omega_p(\rho_{12} - \rho_{21}), \tag{4}$$

$$\dot{\rho}_{43} = -\left[i(\Delta_s - \Delta_c) + \gamma_{43}\right]\rho_{43} - \frac{i\Omega_s}{2}\rho_{23} + \frac{i\Omega_c}{2}\rho_{42}, \tag{5}$$

$$\dot{\rho}_{42} = \left[i\Delta_s - \gamma_{42}\right]\rho_{42} + \frac{i\Omega_s}{2}(\rho_{44} - \rho_{22}) + \frac{i\Omega_c}{2}\rho_{43} + \frac{i\Omega_p}{2}\rho_{41}, \tag{6}$$

$$\dot{\rho}_{41} = -\left[i(\Delta_p + \Delta_s) + \gamma_{41}\right]\rho_{41} - \frac{i\Omega_s}{2}\rho_{21} + \frac{i\Omega_p}{2}\rho_{42}, \tag{7}$$

$$\dot{\rho}_{32} = -\left(\gamma_{32} - i\Delta_c\right)\rho_{32} + \frac{i\Omega_c}{2}(\rho_{33} - \rho_{22}) + \frac{i\Omega_p}{2}\rho_{31}, \tag{8}$$

$$\dot{\rho}_{31} = -\left[\gamma_{31} - i(\Delta_p + \Delta_c)\right]\rho_{31} + \frac{i\Omega_p}{2}\rho_{32} - \frac{i\Omega_c}{2}\rho_{21}, \tag{9}$$

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$$\dot{\rho}_{21} = -(\gamma_{21} - i\Delta_p)\rho_{21} + \frac{i\Omega_p}{2}(\rho_{22} - \rho_{11}) - \frac{i\Omega_c}{2}\rho_{31} - \frac{i\Omega_s}{2}\rho_{41}, (10)$$

$$\rho_{ki} = \rho_{ik}^*, \tag{11}$$

$$\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1, \tag{12}$$

where $\Omega_p = d_{31}E_p/\hbar$, $\Omega_c = d_{32}E_c/\hbar$ and $\Omega_s = d_{42}E_s/\hbar$ are Rabi frequencies of the probe, coupling and signal beam, with d_{ki} is the dipole matrix moments induced on the $|k\rangle$-$|i\rangle$ transition; $\Delta_p = \omega_p - \omega_{31}$, $\Delta_c = \omega_c - \omega_{32}$ and $\Delta_s = \omega_s - \omega_{42}$ are the frequency detunings of the probe, coupling and signal beams; Γ_{ki} is the population decay rate from the level $|k\rangle$ to the level $|i\rangle$, while $\gamma_{ki} = \Gamma_{ki}/2$ is the coherence decay rate.

Under the weak probe field approximation, we have found the solution for the density matrix element ρ_{21} as follows:

$$\rho_{21} = -\frac{i\Omega_p/2}{\gamma_{21} - i\Delta_p + \dfrac{\Omega_c^2/4}{\gamma_{31} - i(\Delta_p + \Delta_c)} + \dfrac{\Omega_s^2/4}{\gamma_{41} - i(\Delta_p + \Delta_s)}}. \tag{13}$$

The total susceptibility for the probe field can be written as [11]:

$$\chi_p = -2\frac{Nd_{21}}{\varepsilon_0 E_p}\rho_{21}$$

$$\equiv \frac{Nd_{21}}{\varepsilon_0 E_p}\frac{i\Omega_p}{\gamma_{21} - i\Delta_p + \dfrac{\Omega_c^2/4}{\gamma_{31} - i(\Delta_p + \Delta_c)} + \dfrac{\Omega_s^2/4}{\gamma_{41} - i(\Delta_p + \Delta_s)}}. \tag{14}$$

with N is the atomic density, ε_0 is the vacuum permittivity.

We now make the Taylor expansion of the expression (14) over Ω_s to extract the linear and nonlinear terms of the susceptibility. We have:

$$\chi = \frac{iNd_{21}^2}{\varepsilon_0 h}\frac{1}{A} - \left[\frac{iNd_{21}^2 d_{42}^2}{\varepsilon_0 h^3}\frac{1}{A^2 B}\right]E_s^2. \tag{15}$$

where:

$$A = \gamma_{21} - i\Delta_p + \frac{\Omega_c^2/4}{\gamma_{31} - i(\Delta_p + \Delta_c)}, \tag{16}$$

$$B = \gamma_{41} - i(\Delta_p + \Delta_s). \tag{17}$$

On the other hand, the total susceptibility (15) can be written in another form as [11]:

$$\chi = \chi^{(1)} + 3E_s^2\chi^{(3)}. \tag{18}$$

with E_s being the electric field of a signal beam.

By comparing Eqs. (15) and (18), we obtain the expressions of first- and third- order susceptibilities for the probe beam as:

$$\chi_p^{(1)} = \frac{iNd_{21}^2}{h\varepsilon_0}\frac{1}{A}, \tag{19}$$

$$\chi_p^{(3)} = \frac{iNd_{21}^2 d_{42}^2}{6h^3\varepsilon_0}\frac{1}{A^2 B}. \tag{20}$$

The linear dispersion n_0 and cross-Kerr nonlinear coefficient n_2 of the four-level Y-type atomic medium for the probe beam are determined by:

$$n_0 = 1 + \frac{\text{Re}(\chi_p^{(1)})}{2}, \tag{21}$$

$$n_2 = \frac{3\text{Re}(\chi_p^{(3)})}{2\varepsilon_0 n_0^2 c}. \tag{22}$$

Here, we use the ^{85}Rb atoms with the $|1\rangle$, $|2\rangle$, $|3\rangle$ and $|4\rangle$ states as $5S_{1/2}(F = 2)$, $5P_{3/2}(F' = 3)$, $5D_{5/2}(F'' = 2)$ and $5D_{5/2}(F'' = 3)$, respectively [19]. The employed parameters as [15]: $N = 10^{12}$ atoms/cm^3; $\Gamma_{32} = \Gamma_{42} = 0.97$ MHz; $\Gamma_{21} = 6$ MHz; $d_{21} = 1.62\times10^{-29}$ C.m; $\omega_p = 3.84\times10^8$ MHz.

First of all, in order to see the enhancement of cross-Kerr nonlinearity n_2 under the EIT condition we plot n_2 versus the probe detuning in two cases of the absence ($\Omega_c = 0$) and presence ($\Omega_c = 3$ MHz) of the coupling beam as in Fig. 2. The dash-dotted line in Fig. 2 represents the EIT spectrum when Rabi frequency $\Omega_c = 3$MHz. We can see from Figure 2 that the nonlinear coefficient is significantly increased on either side of the EIT window at $\Delta_p = 0$. That is to say, a pair of positive-negative peaks of n_2 appears around resonance frequency of the probe field. The amplitude of the nonlinear coefficient in the presence of EIT is increased by several orders compared to that in the absence of EIT.
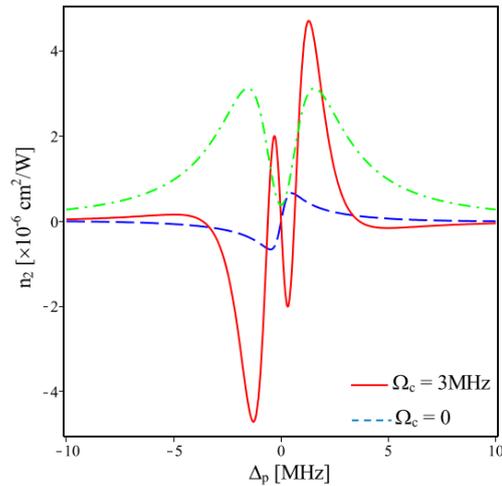
Fig. 2. Variations of the cross-Kerr nonlinear coefficient versus probe detuning Δ_p when $\Omega_c = 0$ (dashed line) and $\Omega_c = 3$MHz (solid line), and $\Delta_c = \Delta_s = 0$. The dash-dotted line represents the EIT spectrum when Rabi frequency $\Omega_c = 3$ MHz.

Next, in Fig. 3 we consider the control of the nonlinear coefficient via the coupling intensity by plotting n_2 versus the coupling Rabi frequency at probe frequency $\Delta_p = \pm2$ MHz, which corresponds to the maximum and minimum points around the EIT window. It is clear that the magnitude and the sign of the nonlinear coefficient are changed with respective to coupling laser intensity.
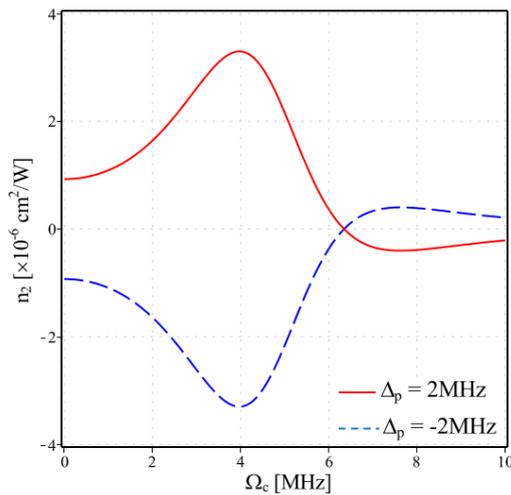
Fig. 3. Variations of the cross-Kerr nonlinear coefficient versus the coupling Rabi frequency Ω_c when $\Delta_p = -2$ MHz (dashed line) and $\Delta_p = 2$ MHz (solid line), and $\Delta_c = \Delta_s = 0$.

Finally, in Fig. 4 we consider the variation of the nonlinear coefficient via coupling frequency detuning Δ_c when $\Delta_p = \Delta_s = 0$ and $\Omega_c = 2$MHz. The variation in Fig. 4 shows that the cross-Kerr nonlinear coefficient of medium also exhibits a pair of positive-negative peaks around $\Delta_c = 0$. The magnitude and sign of the nonlinear coefficient are also controlled concerning the coupling laser frequency.
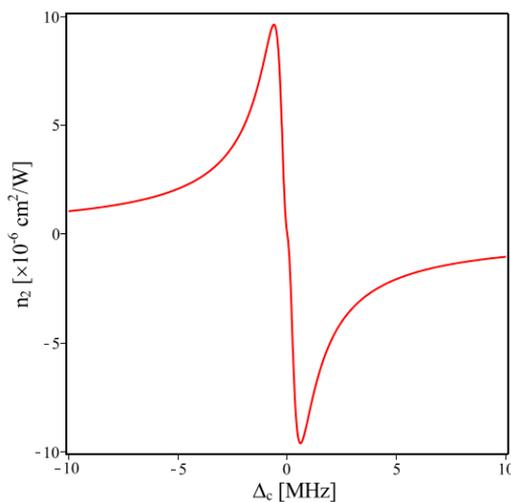
Fig. 4. Variation of the cross-Kerr nonlinear coefficient via coupling detuning Δ_c when $\Omega_c = 3$MHz and $\Delta_p = \Delta_s = 0$.

In conclusions, the enhancement and control of cross-Kerr nonlinearity in a four-level Y-type atomic system have been studied by analytic methods. It is shown that the cross-Kerr nonlinear coefficient of a four-level Y-type atomic system is significantly enhanced around the transparency window. Significantly, the magnitude and sign of the nonlinear coefficient are controlled by tuning the intensity and frequency of the coupling laser. Such an analytical model has an important role to play for experimental observation or implementation of related studies such as slow light, polarization phase gate, all-optical switching and so on.

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