Uniform Scattered Fields from a Parabolic Surface with the Boundary Diffraction Wave Theory

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Abstract—Uniform scattered fields of a cylindrical wave from a parabolic surface are obtained with the theory of the boundary diffraction wave (TBDW). A non-uniform diffracted field is calculated with a regenerated vector potential and rearranged by considering the Fresnel function to obtain a uniform solution. Uniform scattered fields are calculated as the sum of diffracted and geometrical optic fields. Numerical analyses of diffracted and scattered fields in both uniform and non-uniform solutions are in harmony with the literature.

It is Fresnel who explained the diffraction phenomena from the perspective of wave theory in 1818. Fresnel expanded Huygens' study basing on possible interference of waves, which is known as the Huygens-Fresnel principle. Kirchoff revealed the mathematical basics in the following years [1]. The theory of boundary diffraction wave, which is a widely used approach for calculating diffracted fields from aperture systems [2–4], is considered to be the refinement of Young's ideas on the nature of diffraction [5]. The first formulation of the theory of boundary diffraction wave (TBDW) was introduced by Maggi-Rubinowicz considering Young's ideas [6–7]. They independently showed that Helmholtz-Kirchhoff's integral can be converted into a line integral representing edge diffracted fields. The general case of the Maggi-Rubinowicz formulation was expressed by Miyamoto and Wolf for planar and spherical incident fields [8–9].

In this study, the diffracted field of a cylindrical incident field from an opaque parabolic surface is obtained with TBDW for the first time. To our best knowledge, such a solution does not exist in the literature. First, the non-uniform diffracted field is calculated with a regenerated vector potential. Then, the non-uniform result is rearranged by considering the Fresnel integral to obtain a uniform solution.

The time factor e^{jwt} is assumed and suppressed throughout the paper.

For an observation point P, the scalar electrical or magnetic field variation in the Helmholtz-Kirchoff integral can be given as [10]:

$$U(P) = \oiint_S [\nabla_Q \times \vec{W}(Q,P)].\vec{n}dS. \tag{1}$$

The geometry of TBDW can be seen in Fig. 1, where $U(P)$ is the solution of the homogenous Helmhotz-Kirchoff integral. This integral can be divided into two parts by applying the Stokes theorem:

$$U(P) = \int_c \vec{W}(Q,P).d\vec{l} + \sum_i \lim_{\sigma_i \to 0} \int_{c_i} \vec{W}(Q_i,P).\vec{l}dl, \tag{2}$$

here the first term corresponds to the diffracted field, while the second term shows the geometrical optic field. The vector potential in the equation can be given as [11]:

$$\vec{W}(Q,P) = -\frac{U_i(Q) G \sin(\vec{R},\vec{R}_i)\vec{e}_d}{4\pi[1+\cos(\vec{R},\vec{R}_i)]}. \tag{3}$$

The diffraction geometry for an opaque parabolic surface in the domain of a linear current source is given in Fig. 2.

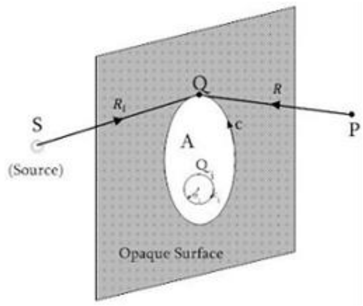

Fig.1. Geometry of the Boundary Diffraction Wave Theory.

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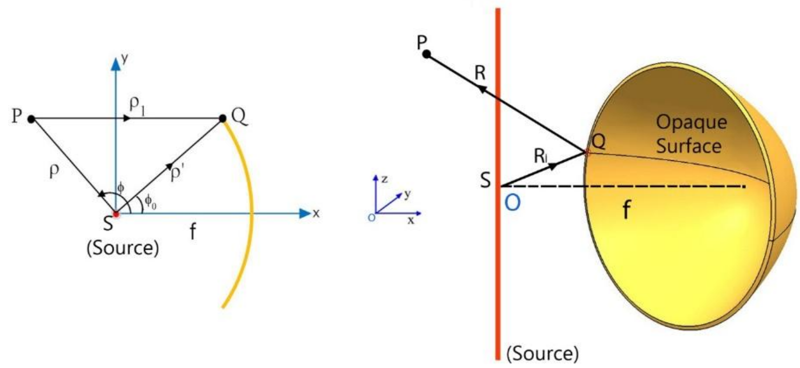

Fig. 2. Diffraction Geometry of the Opaque Parabolic Surface.

As can be seen in Fig. 2, f is the focal length and Q is the diffraction point at which:

$$\rho' = \frac{2f}{1+\cos\phi_0} \tag{4}$$

Here $U_i(Q)$can be written as:

$$U_i(Q) = u_i\,\frac{e^{-jk\rho'}}{\sqrt{k\rho'}} \tag{5}$$

The other parameters in Eq. (3); the Green function is equal to $\frac{e^{-jkR}}{R}$ and"$\vec{e}_d$" is the unit vector in the direction of diffraction. The vector potential can be simplified further by the following equation [12]:

$$\frac{\sin(\vec{R},\vec{R}_i)}{1+\cos(\vec{R},\vec{R}_i)} = \tan\left(\frac{\phi-\phi_0}{2}\right)$$

Rearranging the terms in Eq. (3), the new vector potential can be given as:

$$\vec{\mathrm{W}} = -\vec{e}_z u_i \frac{e^{-jk\rho'}e^{-jkR}}{4\pi\sqrt{k\rho'}R}\tan\left(\frac{\phi-\phi_0}{2}\right). \tag{6}$$

The diffracted field can be easily derived as:

$$U_B(P) = -u_i\frac{1}{4\pi}\tan\left(\frac{\phi-\phi_0}{2}\right)\frac{e^{-jk\rho'}}{\sqrt{k\rho'}}\int_c \frac{e^{-jkR}}{R}dl, \tag{7}$$

by considering Eqs. (1) and (6) for an opaque parabolic surface. Using the Hankel function to obtain a simplified result, the solution can be written as:

$$\mathrm{U_B(P)} = -u_i\frac{1}{2\sqrt{2\pi}}\tan\left(\frac{\phi-\phi_0}{2}\right)\frac{e^{-jk\rho'}}{\sqrt{k\rho'}}\,\frac{e^{-jk\rho_1}e^{-\frac{j\pi}{4}}}{\sqrt{k\rho_1}}. \tag{8}$$

The non-uniform diffracted field solution can be transformed into a uniform result using the Fresnel function [13–14]:

$$\hat{\mathrm{F}}(\xi) = \frac{e^{-j(\xi^2+\pi/4)}}{2\xi\sqrt{\pi}}\ . \tag{9}$$

Here "ξ" is the detour parameter, representing the phase difference between the incident and diffracted fields, which can be written as [15–16]:

$$\xi = -\sqrt{2k\rho_1}\cos\left(\frac{\phi-\phi_0}{2}\right). \tag{10}$$

Using the asymptotic relation

$$\hat{F}(\xi) = F(|\xi|)sgn(\xi), \tag{11}$$

the uniform diffracted field expression can be obtained as

$$U_B(P)$$
$$\cong u_i F(|\xi|)sgn(\xi)e^{jk\rho_1\cos(\phi-\phi_0)}\sin\left(\frac{\phi-\phi_0}{2}\right)\frac{e^{-jk\rho'}}{\sqrt{k\rho'}}\ . \tag{12}$$

Here the Fresnel integral can be given as [17]:

$$F(\xi) = \frac{e^{j\pi/4}}{\sqrt{\pi}}\int_{\xi}^{\infty} e^{-jt^2}dt. \tag{13}$$

The total scattered fields can be given as the sum of the diffracted and incoming fields and can be easily found as:

$$\mathrm{U_{ts}(P) = u_i[\frac{e^{-jk\rho}}{\sqrt{k\rho}}u(-\xi)}$$

$$+F(|\xi|)\mathrm{sgn}(\xi)\, e^{jk\rho_1 \cos(\phi-\phi_0)} \sin\left(\frac{\phi-\phi_0}{2}\right)\frac{e^{-jk\rho'}}{\sqrt{k\rho'}}] \qquad (14)$$

To consider the numerical results, $|u_i| = 1$, $\mathrm{f} = 0.5$ m, incoming surface angle $\phi_0 = \pi/4$ has been chosen. The other parameters are chosen appropriately for real problems such that $k = 8\pi$ and $\rho = 100\pi$ [18–19].

Using the variables in Eq. (8) gives a non-uniform character of the diffracted field as can be seen in Fig. 3. In this figure the diffracted field yields an infinite result at the shadow boundary where $\phi = \pi + \phi_0$.

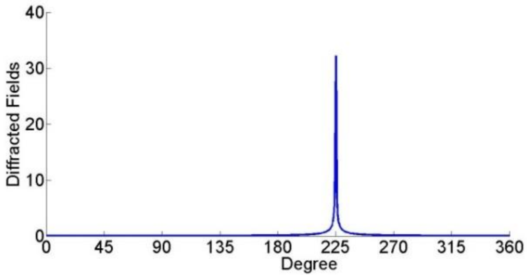

Fig. 3. Non-uniform Diffracted Fields.

Plotting Eq. (12), using the same variables as in Eq. (8), the uniform characteristic of the diffracted field can be seen in Fig. 4. It is obvious that there is no infinite value of the diffracted field.

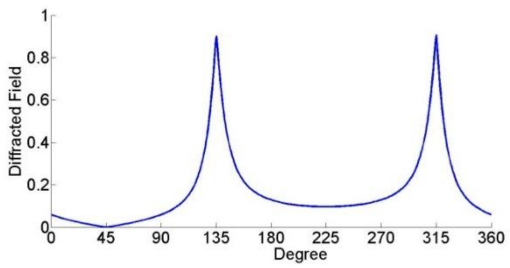

Fig. 4. Uniform Diffracted Fields.

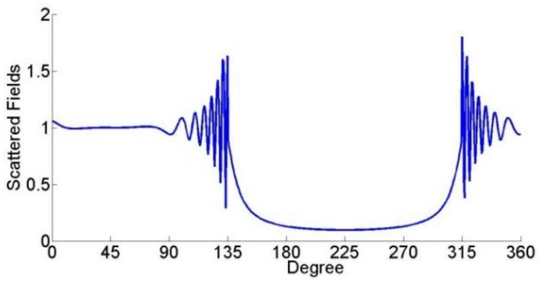

Fig. 5. Uniform Scattered Fields.

Figure 5 demonstrates the variation of total uniform scattered fields, given in Eq. (14), versus the observation angle.

It should be noted that for Fig. 3, Fig. 4 and Fig. 5 the same observation is valid for all the angles of the edge incidence (ϕ_0) and holds true also for all values of ($k\rho$).

The contribution of this study is that a uniform diffracted field from the edge of an opaque parabolic surface is calculated with the boundary diffraction wave theory for the first time. At first, a non-uniform diffracted field is found. Then, the non-uniform result is converted to a uniform shape by using the Fresnel integral. Finally, scattered fields are calculated in a uniform structure. Additionally, numerical analyses of the uniform and non-uniform solutions of diffracted and scattered fields are made in the paper. It can be proved that the non-uniform and the uniform solutions for the diffracted and scattered fields shown in Fig. 3, Fig. 4 and Fig. 5 are in good agreement with the literature [11, 20, 21].

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